4th sem.
1

TRIBHUVAN UNIVERSITY

INSTITUTE OF SCIENCE AND TECHNOLOGY

Final Examination 2076

Subject: **Data Structure and Algorithm**
Course No: **MSCS 251**
Level: **B. Math. Sc. /II Year /IV Semester**

Full Marks: 45
Pass Marks: 18
Time: 2hrs

Candidates are required to give their answer in their own words as far as practicable.
Attempt ALL questions.

Group A [5 × 3 = 15]

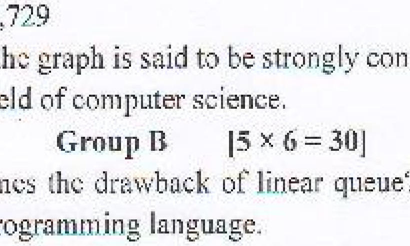
1. Define Stack Data Structure. Write an algorithm for PUSH () and POP () operation for it.
2. Write a simple program to find sum of two number using recursions.
3. Design a binary tree from given traversal:
   Pre-Order: DHIEBFGCA
   In-order :DBHEIAFCG
4. What are the advantages of Binary Search? Search 729 using binary search:
   75,151,203,275,524,591,647,729
5. Define Graph. In which case the graph is said to be strongly connected? Mention some major applications of graph in the field of computer science.

Group B [5 × 6 = 30]

6. How Circular Queue overcomes the drawback of linear queue? Explain how we realize Circular Queue using any high level programming language.
7. What are the basic operations we can perform on list? Explain the algorithm to delete start node and an end node from singly linked list.

   OR

   Write an algorithm to insert a node at the beginning and at any specified position in Doubly Linked List (DLL). Make assumption you need.
8. Define an AVL tree. Construct an AVL tree from the following data:
   64,70,90,1,89,91,79,45,21,16,54, 65
9. Differentiate bubble sort with selection sort. Sort the following list using Merge sort:
   42, 52, 6, 10, 47, 64, 69, 108, 37, 7
10. What is the difference between Kruskal's and Prim's Algorithm? Find the Shortest path from a to z using Dijkstra's algorithm.



OR

Write short note on:
a) ADT
b) Double Ended Queue (Deque)
c) Big 'O' Notation
d) Linked List Representation of Adjacency List

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4th Sem.
2

TRIBHUVAN UNIVERSITY

INSTITUTE OF SCIENCE AND TECHNOLOGY

Final Examination 2076

Subject: **Applied Probability Models** | *Full Marks*: 45
Course No: **MSST 251** | *Pass Marks*: 18
Level: **B. Math. Sc. /II Year /IV Semester** | *Time*: 2hrs

Candidates are required to give their answer in their own words as far as practicable.
Attempt ALL questions.

Group A [5 × 3 = 15]

1. Define simple random sampling. Show that in case of simple random sampling the sample variance is an unbiased estimator of population variance.

2. Let p be the probability of getting head in a single toss of a coin. The coin is tossed 6 times and it is desired to test H_0: p=0.5 against H_1: p=0.67. The H_0 is rejected if more than 3 heads are obtained. Find the probabilities of type I and type II errors. Also, find the power of the test

3. Ministry of Tourism and Civil Aviation has claimed that the average length of stay of tourist in Nepal is 13 days. To test this claim a researcher asked 9 tourists about their length of stay in Nepal and their length of stay in days were 10, 15, 11, 5, 7, 4, 8, 14 and 11. On the basis of this sample result can we conclude that the average length of stay is 13 days?

4. In a survey of 763 women who had started their own businesses, 229 said that they launched their businesses for greater freedom. Only 99 indicated that a desire to make more money drove them to start their businesses.
   a) Set up 90% confidence interval estimate of the population proportion of women who start new businesses to gain freedom.
   b) Set up 99% confidence interval estimate of the population proportion of women who start new businesses to earn more money.

5. For exponential distribution with probability density function $f(x, \beta) = \beta e^{-\beta x}$; x>0 and β >0 based on the random sample of size n.
   a) Find MLE of parameter β.
   b) Find Variance of MLE.
   c) Find the estimate of MLE and its variance if the sample observations are 0.8, 0.2, 1.9, 1.3, 2.8.

Group B [5 × 6 = 30]

6. A company appointed four salesmen A, B, C and D and observed their sales in three seasons. The figures are given below.

Seasons	Salesmen			
	A	B	C	D
Summer	36	36	21	35
Winter	28	29	31	32
Monsoon	26	28	29	29

Test whether there is significant difference in average sales due to (i) different seasons (ii) different salesmen of the company.

7. A population consists of 6 units with values 1, 3, 8, 11, 7 and 4. In sampling without replacement of samples of size 2, show that: $E(\bar{x}) = \mu$, $E(s^2) = S^2$ and $S.E(\bar{x}) = 2.117$.

OR

Define Chi square distribution. Obtain probability density function, mean and variance of chi square distribution with n degree of freedom.

8. If X~ N (0, 1) and Y~N (0, 1) be independent variables then show that U = X/Y follows Cauchy distribution.

9. The manager of newly opened KFC chain of fast food restaurant in Kathmandu derives to test the likeliness of the food with different age-group. Sample of 100 visitors in KFC restaurant were asked their opinion and following is the result.

	20 - 30	30 - 50	>50
Liked	40	20	10
Not liked	10	10	10

Test the hypothesis whether liking of KFC food is associated with age group.

In a sample of 10 observations the sum of square deviation of item from the mean was 120.45. In another sample of 12 observations, the corresponding value was found to be 122.04. Test whether the two samples have the same variance.

OR

A coin is tossed 800 times and heads appear 480 times. Can you infer that the coin is unbiased at 1% level of significance? The monthly advertising expenditure of a company for two products A and B are as follows:

Months	Expenditures in Rs.	
	Product A	Product B
January	100	175
Feb	120	200
March	125	250
April	145	225
May	150	200
June	140	150
July	200	200

Is there sufficient evidence to conclude that the average expenditure on advertising on product B is more than that of product A?

10. A professor is trying to show his students the importance of quizzes even though 90 percent of the final grade is determined by exams. He believes that the higher the quiz grade, the higher the final grade. A random sample of 8 students in his class was selected with the data given below:

Quiz Average	59	92	72	90	95	87	89	97
Final Average	65	84	77	80	77	81	80	83

a) Calculate the correlation coefficient between the two variables and test its significance at 5% level of significance.

b) Calculate the coefficient of determination and explain how much variation in final average is explained by the quiz average?

c) Assuming the linear relationship develops regression equation of final average with Quiz average.

d) Estimate the final average of score of the student whose quiz average is 91.

e) Calculate the standard error of the estimate and interpret the result.

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4th Sem.
3

TRIBHUVAN UNIVERSITY

INSTITUTE OF SCIENCE AND TECHNOLOGY

Final Examination 2076

Subject: **Mathematical Modeling**
Course No: **MSMT 252**
Level: **B. Math. Sc. /II Year /IV Semester**

Full Marks: 45
Pass Marks: 18
Time: 2hrs

Candidates are required to give their answer in their own words as far as practicable.
Attempt ALL questions.

Group A [5 × 3 = 15]

1. Define the first differences of a sequence of numbers. You currently have Rs.5000 in a savings account that pays 0.5% interest each month. You add another Rs.200 each month. Formulate a dynamical system model for this situation. Solve this model by the method of conjecture.

2. What is a mathematical model? Define robust, fragile and sensitivity of a model. Give the geometric interpretation of $y \propto x$. Also state any three famous relations using proportionalities.

3. Fit the model $y = ca^x$ to the following data set using transformed least square method:

x	1	2	3	4
y	8.1	22.1	60.1	165

4. Use Golden Section Search Method to maximize $f(x) = -3x^2 + 21.6x + 1$ in the interval $0 \le x \le 25$ with a tolerance of $t = 0.25$.

5. When a critical point is said to be stable? Determine the nature of the critical point (0, 0) of the autonomous system $\frac{dx}{dt} = 2x - 7y$, $\frac{dy}{dt} = 3x - 8y$ and determine whether or not the point is stable.

Group B [5 × 6 = 30]

6. The following data were obtained for the growth of a sheep population introduced into a new environment on the island of Tasmania.

Year:	1814	1824	1834	1844	1854	1864
Population:	125	275	830	1200	1750	1650

Plot the data. Is there a trend? Plot the change in population versus years elapsed after 2012. Formulate a discrete dynamical system that reasonably approximates the change you have observed. Also fit your model to the original data.

OR

A car rental company has distributorship in Orlando and Tampa. The company specializes in catering to travel agents who want to arrange tourist activities in both cities. Consequently, a traveler will rent a car in one city and drop the car off in the second city. Travelers may begin their itinerary in either city. The historical records reveal that 40% of the cars rented in Orlando are returned to Orlando, whereas 60% end up in Tampa. Of the cars rented from the Tampa office, 30% are returned to Tampa, whereas 70% end up in Orlando. Write a dynamical system model for this problem. Find the equilibrium value. Is the model sensitive to initial values?

7. Construct a mathematical model for predicting the weight of the Terror Bird as a function of the circumference of its femur where the assumptions are:
a) Terror birds are geometrically similar to other large birds
b) Constant average density.

Femur circumference (cm)	Body weight (kg)
0.7943	0.0832
0.7079	0.0912
1.000	0.1413
1.1220	0.1479
1.6982	0.2455
1.2023	0.2818
1.9953	0.7943
2.2387	2.5119
2.5119	1.4125
2.5119	0.8913
3.1623	1.9953
3.5481	4.2658
4.4668	6.3096
5.8884	11.2202
6.7608	19.95
15.136	141.25
15.85	158.4893

8. Use Chebyshev criterion to fit the model y=ax to the data set given below:

x	1	2	3
y	2	5	8

9. Define a linear program. When an optimization problem is said to be a goal program? Maximize $Z=2x - y$, subject to $x + y \geq 5$, $-x + y \leq 1$, $5x + 4y \leq 40$, $x \geq 0$, $y \geq 0$ by using Big M Method:

10. Imagine a small pond that is mature enough to support wildlife. We desire to stock the pond with game fish, say trout and bass. Let x(t) denotes the population of the trout at any time t, and let y(t) denotes the bass population. Is coexistence of the two species in the pond possible? If so, how sensitive is the final solution of population levels to the initial stockage levels and external perturbations? Assume that, initially, the environment can support an unlimited number of trout and there is a competition of trout with bass population for living space and a common food supply. The effect of the bass population is to decrease the growth rate of the trout population and this decrease is approximately proportional to the number of possible interactions between the two species. The situation for bass population is same as trout.

OR

Construct a predator-prey model for baleen whales and Antarctic krill .The whales eat krill, and krill live on the plankton in the sea. If the whales eat so many krill that the krill cease to be abundant, the food supply of the whales is greatly reduced. Then the whales will starve or leave the area in search of a new supply of krill. As the population of baleen whales dwindles, the krill population makes a comeback because not so many of them are being eaten. As the krill population increases, the food supply for the whales grows and, consequently, so does the baleen whale population. Also, more baleen whales are eating increasingly more krill again. In the pristine environment, does this cycle continue indefinitely or does one of the species eventually die out? Assume that ocean can support an unlimited number of krill and the growth rate of the krill is diminished in a way that is proportional to the number of interactions between them and the baleen whales. Analyze the model graphically.

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4th Sem.
4

TRIBHUVAN UNIVERSITY

INSTITUTE OF SCIENCE AND TECHNOLOGY

Final Examination 2076 (Partial)

Subject: **Mathematical Statistics** | *Full Marks*: 45
Course No: **MSST 252** | *Pass Marks*: 18
Level: **B. Math. Sc. /II Year /IV Semester** | *Time*: 2hrs

Candidates are required to give their answer in their own words as far as practicable.
Attempt ALL questions.

Group A $[5 \times 3 = 15]$

1. For each of the following process
   a) General Random Walk b) Markov Chain c) Markov Jump Process
   State whether the state space and time set is discrete or continuous or either.

2. Define how the following forms of censoring arise in a survival investigation:
   a) Right Censoring b) Type I Censoring c) Random Censoring
   Explain which type of the censoring (Right censoring, Type I, Random) is present in the following situations: a) For the members who change employment b) For the members who retired c) The policy is not renewed.

3. Derive from first principles the forward version of the Kolmogorov differential equations:
   $\frac{\partial}{\partial t}P(s, t) = P(s, t)\, A(t)$, where A is the matrix of transition rates.

4. Explain why the criteria of smoothness and adherence to data both need to be met when performing a graduation of mortality data that will be used to set premium rates. And hence, describe the smoothness test, stating any relevant formulae.

5. A mortality investigation was held between 1 January 2007 and 1 January 2009. The following information was collected. The figures in the table below are the numbers of lives on each census date with the specified age labels.hat do you mean by principle of correspondence? List the requirements for exact estimation of central exposed to risk with regard to mortality investigation.

Age last birthday	1.1.07	1.1.08	1.1.09
48	3486	3384	3420
49	3450	3507	3435
50	3510	3595	3540

During the investigation there were 42 deaths at age 49 nearest birthday. Estimate μ_{49} stating any assumptions that you make.

Group B $[5 \times 6 = 30]$

6. An insurance company operates a no claims discount system with discount levels of 0%, 30%, 40%, 50% and 60%. The rules are as follows:
   At the end of a claim free year, a policyholder moves up one level (or remains on the maximum discount level).
   At the end of a year in which exactly one claim was made, a policyholder drops back two levels (or moves to zero discount).

At the end of a year in which more than one claim was made, a policyholder drops back to zero discounts.

For a particular driver in any year, the probability of a claim free year is 0.7, the probability of exactly one claim is 0.2, and the probability of more than one claim is 0.1.Write down the transition matrix for this time-homogeneous Markov chain. If a large number of people having the same claims distribution take out policies at the same time, calculate the proportion you would expect to be in each discount category in the long run.

7. A pension scheme only allows retirement at exact age 65. An investigation of the mortality of the retired members of the scheme was carried out over the period 1 January 2001 to 31 December 2006.ortality table has been estimated for the ages 4 to 100 inclusive. The rates have been graduated fitting a mathematical formula to the crude estimates. The deviations of the observed number of deaths from the expected number of deaths at each age using the graduated mortality rates have been calculated. The results are following:

Member	Date of Retirement	Date of Death (if occurred during the investigation period)
1	1 April 1998	30 April 2005
2	1 August 2000	
3	1 February 2001	
4	1 June 2002	31 August 2004
5	1 August 2002	31 December 2006
6	1 March 2004	
7	1 May 2004	30 November 2006
8	1 January 2005	

All months should be assumed to be of equal length. Calculate the Kaplan-Meier (product-limit) estimate of the survival function $S\ t\ (65)$ from these data, stating clearly any additional assumptions that you make.

OR

Calculate the Nelson –Aalen estimate of the Risk function $F\ t\ (65)$ from these data, stating clearly any additional assumptions that you make.

8. A study is undertaken by railway authorities in respect of the new train engines that they are planning to buy. The train engines being used in a country A are observed for a month and details pertaining to their operating hours, maintenance jobs, In-Yard hours (i.e. not on duty hours) etc are being captured. The authorities have observed the following:

A perfectly working Engine moves to a Yard after it has been in operation for a given time interval.

An Engine currently resting in a Yard is called upon for operation after specific time intervals. If an Engine either currently in Operation or put into Operation from Yard develops any fault then is moved to a maintenance facility for repairs. After successful repairs, the engine moves to Yard before being pressed into Operation.

If the fault cannot be resolved the engines are retired from further service.

a) Draw a diagram illustrating a multiple-state model which the investigators could use to make their estimates, using the four states: "In-Yard", "Operation", "Maintenance" and "Retired".

b) Derive from first principles the Kolmogorov differential equation for "Operation" state.

OR

c) Write down the likelihood of the data in terms of the waiting times in each state, the numbers of transitions of each type, and the transition intensities, assuming the transition intensities are constant.

d) Derive the maximum likelihood estimator of the rate "Operation" and "Maintenance" state.

9. Cox proportional hazards model was used to study the mortality experience amongst group of lives classified according to smoker or non smoker and male or female. There are two covariates; $z_1=1$ for smoker and $z_1=0$ for non smoker and $z_2=1$ for male $z_2=0$ for female, The parameter estimates are $\beta_1 = 0.05$ and $\beta_2 = 0.15$.

   a) Clearly mention for which the baseline hazard applies.

   b) Determine the relative risk of a female non-smoker compared to male smoker.

10. A life insurance company has graduated its own mortality experience for term assurance business over the past 15 years against a standard table:

Age Group	Exposed to Risk	Graduated Rates	Actual Death
50	24584	0.000605	14
51	32587	0.000683	32
52	15784	0.000748	16
53	21336	0.000823	22
54	25874	0.000908	24
55	21554	0.001005	22
56	23967	0.001104	25
57	25811	0.001239	30
58	26911	0.001378	28
59	28445	0.001536	38
60	30205	0.001713	45

a) Carry out a test for overall goodness of fit of the data, using a 95% significance level.

b) Further, carry out one additional test to more appropriate conclusion, and give your conclusion.

For each test you should clearly indicate the Null Hypothesis and Findings of your test.

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4th sem.
5

TRIBHUVAN UNIVERSITY

INSTITUTE OF SCIENCE AND TECHNOLOGY

Final Examination 2076

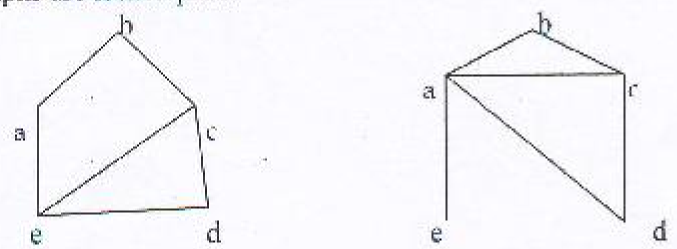
Subject: **Discrete Mathematics**
Course No: **MSMT 253**
Level: **B. Math. Sc. /II Year /IV Semester**

Full Marks: 45
Pass Marks: 18
Time: 2hrs

Candidates are required to give their answer in their own words as far as practicable.
Attempt ALL questions.

Group A [5 × 3 = 15]

1. What are the negations of the statement $\forall x\,(x^2 > x)$ and $\exists x\,(x^2 = 2)$? Show that $\neg\forall x\,(P(x) \rightarrow Q(x))$ and $\exists x\,(P(x) \wedge \neg Q(x))$ are logically equivalent.
2. Define greatest common divisor and least common multiple of two integers. Let a and b be two positive integers, then prove that ab =gcd(a,b).lcm(a,b). Also, express gcd (252,198) as a linear combination of 252 and 198.
3. State principle of mathematical induction. Use mathematical induction to prove that $2^n < n!$ for every positive integer n with $n \geq 4$.
4. State binomial theorem for a non negative integer n. What is the coefficient of $x^{12}y^{13}$ in the expansion of $(2x-3y)^{25}$?
5. Define bipartite graph. Why K_3 is not bipartite? Also determine whether the following pair of graphs are isomorphic:



Group B [5 × 6 = 30]

6. State and prove Associative laws of set algebra by using membership table. Define a poset. Show that the divisibility relation on the set of positive integers is a poset. When a poset is said to be totally ordered? Give an example of totally ordered set.

OR

What is an equivalence relation on a set A? Let $R = \{(x, y): x, y \in N,\ x - y \text{ is divisible by } 5\}$ be a relation. Prove that R is an equivalence relation. Also show that the set of odd positive integers is a countable set.

7. Write the steps of strong induction. Suppose we can reach the first and second rungs of an infinite ladder, and we know that if we can reach a rung, then we can reach two rungs higher. Can we prove that we can reach every rung using the principle of mathematical induction? Can we prove that we can reach every rung using strong induction?

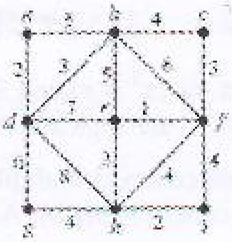
OR

What is the difference between strong induction and principle of mathematical induction? Use strong induction to show that if n is an integer greater than 1, then n can be written as the product of primes.

8. Telephone numbers consists of 7 digits and none of them begin with zero. How many telephone numbers could be possible if no digit appears more than once? If the successive coefficients in the expansion of $(1+x)^n$ are 28, 56 and 70, find n. Show that the coefficient in the middle term of the expansion of $(1+x)^{2n}$ is equal to the sum of the coefficients in the two middle terms of the expansion of $(1+x)^{2n-1}$.

9. State Fermat's little theorem. Show that 341 is a pseudoprime to the base 2. Find all solutions to the system of congruence $x \equiv 7 \pmod 9$, $x \equiv 4 \pmod{12}$, $x \equiv 16 \pmod{21}$.

10. Define a rooted tree. When it is called m-ary and full m-ary? Define spanning tree and minimum spanning tree. Use Kruskal's algorithm to find a minimum spanning tree in the graph given below:



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