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A Hesitant Fuzzy Envelope Based Expert System in Human Decision Making

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Abstract: *Naturally, individual decision style is qualitative rather than quantitative settings. In nature, the human way of thinking is uncertain and fuzziness that demands the use of the linguistic approach of problems related to the decision. The group decision making process is highly affected by hesitant situations among the members for clarity-based decisions. In order to remove the hesitant situations, the proposed Hesitant Fuzzy Envelope expert system provides the group decision making processes with more realistic output in envelope form rather than CRISP one.In this study, we shall discuss a linguistic-based expert system that will help to make more realistic decisions in a hesitant situation by using Hesitant Fuzzy Envelope technique.*

Keywords: *Expert systems, Human decision making, Hesitant Fuzzy envelope, Soft computing*

1. Introduction

In human beings, decision making, is a natural process, based on mental and reasoning processes in the uncertain, imprecise, and vague environment [5]**.** To model this type of decision-making problem, we need linguistic information that comes from experts to express their knowledge in qualitative value.

The linguistic model has successfully provided for managing the complexity of real-world problems. Most of the linguistic model make a decision based on a single linguistic term. But in a high degree of uncertainty depends on a linguistic context where the experts have hesitated among the different linguistic terms.

There have different types of literature to provide linguistic context rather than a single linguistic term. Merging different types of linguistic terms into a single word, Ma et al. [6] increase the flexibility of the linguistic model. By using logical connectives, Tang and Zheng [9] build a linguistic model to manage linguistic expressions. Rodriguez et al. [7] has introduced the Hesitant Fuzzy Linguistic term set (HFLTS) to improve the importance of the linguistic decision-making model. Most of the Hesitant Fuzzy Linguistic model operates with a symbolic model, and the final output gives CRISP values, so initial fuzzy representation has lost in outcomes. So, the aim of this present study is that developed an expert system using the Fuzzy Envelop concept in HFLTS where the results in this decision-making model come in the form of Fuzzy membership function.

The present article is structured as follows: Section 2 discusses some preliminary concepts. Section 3 discusses about Hesitant Fuzzy Envelop. Section 4 introduces an expert system using Hesitant Fuzzy Envelop concept. Section 5 deals with Validation and Advantage of the proposed expert system. Section 6: Draw a proper conclusion.

2. Preliminaries

2.1 Linguistic Variable [11]

Definition 1: A linguistic variable is defined by a quintuple $(H, T(H), U, G, M)$ in which H is name of the variable, $T(H)$ is the term set of H, U is the universe of discourse, G is a syntactic rule which generates $T(H)$, M is a semantic rule associated with each linguistic value.

It is necessary to choose a linguistic descriptor to deal with a linguistic variable. For selecting linguistic descriptor, there have various kind of approaches. But in our cases,

Let $S = \{s_0, s_1, s_2, \cdots, s_{\#S}\}$ be a set of linguistic terms satisfied

1. $s_i \leq s_j$ iff $i \leq j$
2. Negation $(s_i) = s_{\#S-i}$
3. $\max(s_i, s_j) = s_i$ and $\min(s_i, s_j) = s_j$ if $i \geq j$.

2.2 A Context-free grammar [1] [2]

A context-free grammar(G) is used for generating linguistic expressions utilizing the set of linguistic terms.

Definition 2: G, mainly defined in 4-tuple (V_N, V_T, I, P) where V_N: Non-terminal symbols, V_T: Terminal symbols, I: Starting symbols, : Production Rules

P = {(Primary Term) or (Unary Relation) (Primary Term)
or (Binary Relation) (Primary Term) (Conjunction) (Primary Term)}

(Primary Term) = $s_i \in S$

(Unary Relation) = $Lower\ Than\ or\ Greater\ Than$

(Binary Relation) = $Between$

(Conjunction) = And

2.3 Hesitant Fuzzy Linguistic Term Set (HFLTS) [7]

Definition 3: Let $S = \{s_0, s_1, \cdots, s_{\#S}\}$ be a linguistic term set, an HFLTS, H_S, is an ordered finite subset of the consecutive linguistic terms of S.

$$\therefore H_S = \{s_i, s_{i+1}, \cdots, s_j | s_i, s_{i+1}, \cdots, s_j \in S\}.$$

Transformation Function: The function E_G transform a linguistic expression S_{ll} which is generated by a context-free grammar G into an HFLTS H_S.

$$\therefore E_G: S_{ll} \rightarrow H_S.$$

2.4 ORness and ANDness operators [10]

Let $A = [a_1, a_2, \cdots, a_n]$ be a vector of n elements and $B = [b_1, b_2, \cdots, b_n]$ where $b_j$is the jth largest element of A. $W = [w_1, w_2, \cdots, w_n]^T$ be the associated weight vector satisfying $w_i \in [0,1], i = 1,2, \cdots, n$ and $\sum_{i=1}^{n} w_i = 1$. Then ordered weighting average (OWA) operator on A is defined by

$$OWA_W(A) = B.W$$

Definition 4: The ORness operator corresponding to the weighting vector W is also an OWA operator and defined as

$$ORness(W) = \frac{1}{n-1} \sum_{i=1}^{n} w_i (n - i)$$

Where $0 \leq ORness(W) \leq 1$, and $ANDness(W)$ is defined by $ANDness(W) = 1 - ORness(W)$.

Optimistic OWA operator or OR like OWA operator are those whose $ORness(W) > 0.5$ and pessimistic OWA or AND like OWA operator are those whose $ANDness(W) < 0.5$.

3. Hesitant Fuzzy Envelope

Let $S = \{s_0, s_1, s_2, \cdots, s_{\#S}\}$ be a set of linguistic terms and H_S be an HFLTS on S such that

$$H_S = \{s_i, s_{i+1}, \cdots, s_j | s_i, \cdots, s_j \in S\}.$$

Also, let each s_i in S is defined in the form of a triangular membership function $T(a, b, c)$ [11]

Hesitant Fuzzy Envelope calculation is performed into three steps:

3.1 Finding the elements

Let $A^k = T(a_L^k, a_M^k, a_R^k), k = 0,1, \cdots, \#S$ be membership function of $s_k \in S$.

But according to fuzzy partition [8] $a_R^{k-1} = a_M^k = a_L^{k+1}$

So, the points of all the linguistic terms in H_S is

$$T(H_S) = \{a_L^i, a_M^i, a_M^{i+1}, a_M^{i+2}, \cdots, a_M^j, a_R^j\}$$

3.2 Compute the parameters of the membership function for fuzzy envelope

The fuzzy envelope of H_S can be represented by the trapezoidal membership function $T(a, b, c, d)$ which can calculate from the points of $T(H_S)$ where

$$a = min\{a_L^i, a_M^i, a_M^{i+1}, a_M^{i+2}, \cdots, a_M^j, a_R^j\}$$

$$d = max\{a_L^i, a_M^i, a_M^{i+1}, a_M^{i+2}, \cdots, a_M^j, a_R^j\}$$

For computing b and c, there arise two cases:

a) If $i + j = Odd$

i. If $i + 1 = j$

$b = a_M^i$ and $c = a_M^{i+1}$

ii. If $i + 1 < j$

$$b = OWA_{W^s}\left(a_M^i, a_M^{i+1}, a_M^{i+2}, \cdots, a_M^{\frac{i+j-1}{2}}\right)$$

$$c = OWA_{W^t}\left(a_M^j, a_M^{j-1}, a_M^{j-2}, \cdots, a_M^{\frac{i+j+1}{2}}\right)$$

b) If $i + j = Even$

$$b = OWA_{W^s}\left(a_M^i, a_M^{i+1}, a_M^{i+2}, \cdots, a_M^{\frac{i+j}{2}}\right)$$

$$c = OWA_{W^t}\left(a_M^j, a_M^{j-1}, a_M^{j-2}, \cdots, a_M^{\frac{i+j}{2}}\right)$$

Where the weighting vector W^s and W^t are computed from the next step.

3.3 Compute the weighting vector W^s and W^t

Let $\alpha \in [0,1]$ be a parameter then through the formula we have calculated W^s and W^t vectors:

$W^s = [w_1^s, w_2^s, w_3^s, \cdots, w_n^s]^T$, where $w_1^s = \alpha^{n-1}; w_i^s = (1 - \alpha)\alpha^{n-i}, i = 2,3, \cdots, n$

and

$W^t = [w_1^t, w_2^t, w_3^t, \cdots, w_n^t]^T$, where $w_1^t = \alpha; w_i^t = \alpha(1 - \alpha)^{i-1}, i = 2,3, \cdots, n.$

4. Expert System

Let in an expert system there have n alternatives $A = \{a_1, a_2, \cdots, a_n\}$ and m criteria $C = \{c_1, c_2, \cdots, c_m\}$. With the help of expert panels, for each alternative a_i corresponding to each criterion c_j, we have built Hesitant Fuzzy Linguistic Matrix $HFLM = [H_S^{ij}]_{n\times m}$ of order $n \times m$ from a linguistic term set $S = \{s_0, s_1, s_2, \cdots, s_{\#S}\}$.

Now the expert system is executed through the following steps:

4.1 Convert HFLM elements to the Fuzzy Envelope

In this step, we have converted each $HFLTS\ H_S^{ij}; i = 1,2,\cdots,n$ and $j = 1,2,\cdots,m$ into a fuzzy envelope through the steps, discussed in section 3.

Let fuzzy envelope of $H_S^{ij} = T(a^{ij}, b^{ij}, c^{ij}, d^{ij})$

4.2 Averaging the fuzzy envelopes for each alternative

From the previous steps, we have got m fuzzy envelopes (corresponding to each criteria) for each alternative a_i.

Now we have averaging these m fuzzy envelopes for each a_i using the formula.

$$T(a_i) = T\left(\frac{1}{m}\sum_{j=1}^{m}(a^{ij}, b^{ij}, c^{ij}, d^{ij})\right)$$
$$= T(a^i, b^i, c^i, d^i)\ , i = 1,2\cdots,n$$

4.3 Preference Relation between alternatives

In this step, we have built the preference relation **[4]** matrix $P = [p_{ij}]_{n\times n}$ by distance measuring between each pair of alternatives $(a_i, a_j), i,j = 1,2\cdots,n$. Distance measuring between two fuzzy envelopes **[3]** are as follows

$$p_{ij} = d\left(T(a_i), T(a_j)\right)$$
$$= \tfrac{1}{4}\left(|a^i - a^j|^p + |b^i - b^j|^p + |c^i - c^j|^p + |d^i - d^j|^p\right)^{\frac{1}{p}}, if\ 1 \le p < \infty, i \ne j$$

4.4 Calculate non-dominance degree

The non-dominance degree **[4]** NDD_i for each alternative $a_i, i = 1,2,\cdots,n$ is calculated by

$$NDD_i = min\{1 - p_{ji} | j \ne i, j = 1,2,\cdots,n\}$$

4.5 Finding the best alternatives

According to the maximum value of the non-dominance degree NDD_i for alternative a_i calculated the best alternative.

So,

$$A^{Best} = \left\{a_i \middle| NDD_i = Max_{a_j\in A}(NDD_j)\right\}$$

5. Validation and Advantage

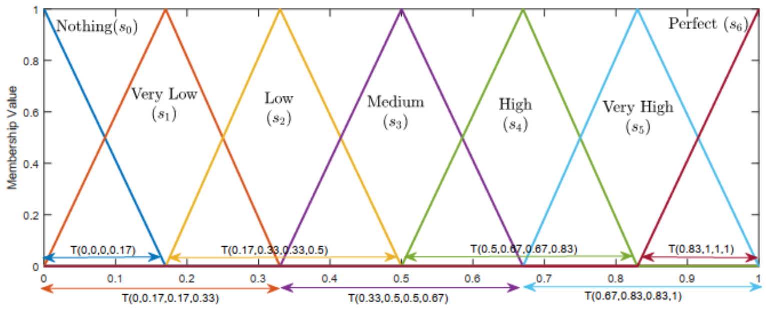
5.1 Validation

To validate the proposed expert system, we have taken the example which has given in **[7].**

Let there have three alternatives $X = \{x_1, x_2, x_3\}$ and three criteria

$$C = \{c_1, c_2, c_3\} \text{ and } S = \{s_i | i = 0,1,\cdots,6\}$$

be the set of linguistic terms whose details has been shown in figure 1.



The assessment values for each alternative x_i corresponding to each criterion c_j are given the form of HFLM and its corresponding Fuzzy Envelopes are shown in table 1.

Table 1 Assessment Values of HFLM and its corresponding Fuzzy Envelopes

H_S^{ij}	c_1	c_2	c_3
x_1	$\{VL, L, M\}$ $T(0,0.30,0.36,0.67)$	$\{H, VH\}$ $T(0.50,0.67,0.83,1)$	$\{H\}$ $T(0.5,0.67,0.67,0.83)$
x_2	$\{L, M\}$ $T(0.17,0.33,0.50,0.67)$	$\{M\}$ $T(0.33,0.5,0.5,0.67)$	$\{N, VL, L\}$ $T(0,0,0.15,0.5)$
x_3	$\{H, VH, P\}$ $T(0.5,0.85,1,1)$	$\{VL, L\}$ $T(0,0.17,0.33,0.5)$	$\{H, VH, P\}$ $T(0.5,0.85,1,1)$

After calculating Fuzzy envelops we aggregate it for each alternative x_i according to the step 2 in section 4.

$$T(x_1) = T(0.33,0.55,0.62,0.83)$$

$$T(x_2) = T(0.17,0.28,0.38,0.61)$$

$$T(x_3) = T(0.33,0.62,0.78,0.83)$$

Here we have taken $p = 1$ for calculating distance between the different pair of alternatives. So, the preference relation matrix P will be

$$P = \begin{pmatrix} NaN & 0.2225 & 0.0575 \\ 0.2225 & NaN & 0.2800 \\ 0.0575 & 0.2800 & NaN \end{pmatrix}$$

The non-dominance degree for each alternative is

$NDD_1 = min\{1 - 0.2225, 1 - 0.0575\}$ =0.775

NDD_2 =0.7200

NDD_3 =0.7200

So, through our proposed system we have found that 1st alternative i.e. x_1 is the best among the three alternatives, which is the same result of [7].

5.2 Advantage

The main advantage of the proposed expert system is to remove hesitant phase in group decision making process. The output of the system appears in Envelope form.

6. Conclusion

Most of the time expert provide their assessments by using single linguistic term. But in hesitant situation, experts needed to provide more precious linguistic expressions. Here HFLTS provide to increase flexibility in an indecisive case. For a decision-making system, an envelope for HFLTS is used as a linguistic interval, where as in the final result initial fuzziness loosed.

In this present study, the expert system is designed through the concept of fuzzy envelopes where the initial fuzzy representation of the linguistic terms is aggregate to fuzzy membership function without loss of the initial fuzziness. Invalidation processes using the same example and got the same result which has used in the decision-making model through the symbolic linguistic interval envelopes. But the advantage is that our output may not lose the initial fuzziness.

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