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Statistical Convergence Estimates for *(p, q)*-Baskakov-Durrmeyer Type Operators

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Abstract: *This paper concerns with the study of(p, q)-analogue of genuine Baskakov- Durrmeyer type operators. We establish the direct approximation theorem, a weighted approximation theorem followed by the estimations of the rate of convergence of these operators for functions of polynomial growth on the interval* $[0, \infty)$.

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1. Introduction

In the theory of approximation, the quantum calculus has been studied for a long time. Quantum calculus was started by the well known mathematician Lupus [11], when he first proposed the q-variant of the Bernstein polynomials. T. Kim gave his contribution on q-type of polynomial in [9], [10]. In the same notions similar type of results on q-analogue of linear positive operators were obtained by [19], [20], [21] etc. Present paper deals with *(p, q)*-calculus (post-quantum calculus), which is an advanced extension of quantum calculus. Mursaleen et al. [12], introduced the Bernstein polynomials using (p, q)-calculus, which was further improved in [13].

(p, q)-calculus was introduced by the classical work of Sahai and Yadav [25]. Recently, a lot of work on *(p, q)*-version of linear positive operators has been published in Acer et al. [2] [1], Aral and Gupta [5], Gupta [8], Mursaleen et al. [14] [15]. We also consider some more results on approximation of functions by positive linear operators using *(p, q)*-calculus given in ([3],[16],[17]).

P. Maheshwari and M. Abid [18] published a paper on approximation of *(p, q)* Szasz-Beta-Stancu operators. To recall some definition and notations of *(p, q)*-calculus, we refer to authors [22], [23] and [24].

The (p, q)-number is defined as

$$[n]_{p,q} = \frac{p^n - q^n}{p - q}, \qquad n = 0, 1, 2 \dots and \ [0]_{p,q} = 0.$$

The *(p, q)*-factorial $[n]p, q!$ is defined as

$$[n]p, q! = \prod_{k=1}^{n} [k]_{p,q}, \qquad n \geq 1, \ [0]p, q! = 1.$$

The *(p, q)*-binomial coefficient is given by

$$\begin{bmatrix} n \\ k \end{bmatrix}_{p,q} = \frac{[n]p, q!}{[n-k]p, q! \, [k]p, q!}, \ 0 \leq k \leq n.$$

(p, q)-derivative is given as

$$D_{p,q} f(x) = \frac{f(px) - f(qx)}{(p-q)x}, \ x \neq 0.$$

The *(p, q)*-power basis is defined below

$$(x \oplus y)_{p,q}^{n} = (x+y)(px+qy)(p^2x+q^2y)\dots(p^{n-1}x+q^{n-1}y)$$

$$(x \ominus y)_{p,q}^{n} = (x-y)(px-qy)(p^2x-q^2y)\dots(p^{n-1}x-q^{n-1}y).$$

We propose *(p, q)*-Beta function as,

$$B_{p,q}(m,n) = p^{n/2}q^{m/2}\int_0^{\infty/A}\frac{x^{m-1}}{(1+\oplus x)_{p,q}^{n+m}}d_{p,q}t, \quad m,n \in N, \tag{1}$$

(p,q)-Gamma function is defined as

$$\Gamma_{p,q}(n+1) = \frac{(p\ominus q)_{p,q}^{n}}{(p-q)^n} = [n]_{p,q}!, \quad 0<q<p. \tag{2}$$

Proposition 1. [23] The *(p, q)*-integration by parts is given by

$$\int_a^b g(px)\,D_{p,q}h(x)d_{p,q}x = g(b)h(b) - g(a)h(a) - \int_a^b h(qx)\,D_{p,q}g(x)d_{p,q}x.$$

The *(p, q)*-Beta function of second kind [5] is given by

$B_{p,q}(m,n) = p^{n/2}q^{m/2}\int_0^{\infty}\frac{x^{m-1}}{(1+\oplus px)_{p,q}^{n+m}}d_{p,q}x,$ where $m,n \in N$.

The relation between *(p, q)*-Beta and *(p, q)*-Gamma functions is given as

$$B_{p,q}(m,n) = q^{\frac{2-m(m-1)}{2}}p^{\frac{-m(m+1)}{2}}\frac{\Gamma_{p,q}m\Gamma_{p,q}n}{\Gamma_{p,q}(m+n)}.$$

To approximate Lebesgue integrable function on the interval $[0,\ \infty)$, Agrawal and Thamar [4] introduced the following operators, which is an extension of Srivastava-Gupta operators [25],

$$G_n(f,x) = (n-1)\sum_{k=1}^{\infty}s_{n,k}(x)\int_0^{\infty}s_{n,k-1}(t)f(t)dt + p_{n,0}\,f(0), \tag{3}$$

where

$$s_{n,k}(x) = \begin{bmatrix} n+k-1 \\ k \end{bmatrix}\frac{x^k}{(1+x)^{n+k}}.$$

We introduce the *(p, q)*-analogue of genuine Baskakov-Durrmeyer operators for$x \in [0,\infty)$ and $0 < q < p \le 1$,the operators are defined as

$$G_n^{p,q}(f,x) = [n-1]_{p,q}\sum_{k=1}^{\infty}s_{n,k}^{p,q}(x)p^{(n-1)^2+k}q^{k(k-1)} \times \int_0^{\infty/A} s_{n,k-1}^{p,q}(t)f(t)d_{p,q}t + p_{n,0}^{p,q}(x)\,f(0) \tag{4}$$

where $s_{n,k}^{p,q}(x) = \begin{bmatrix} n+k-1 \\ k \end{bmatrix}_{p,q}\frac{x^k}{(1\oplus x)_{p,q}^{n+k}}$.

It can be noted here, if we put *p = q = 1*, we get well known Baskakov Durrmeyer operators.

2. Auxiliary Results

In this section, we establish some basic results to prove our main theorems.

Lemma 1. For $x \in [0,\infty)$ and $0<q<p\le 1$, we have

$$G_n^{p,q}(1,x) = 1$$

$$G_n^{p,q}(t,x) = \frac{[n]_{p,q}x}{pq[n-2]_{p,q}}$$

$$G_n^{p,q}(t^2,x) = \frac{[n]_{p,q}x^2}{p^2q^4[n-2]_{p,q}[n-3]_{p,q}} + \frac{[n]_{p,q}[2]_{p,q}x}{p^{-n+4}q^3[n-2]_{p,q}[n-3]_{p,q}}.$$

Proof. By the definition *(p, q)*-Beta function given in (1), we get the following estimates

(i) For *f(t)* = 1, we have

$$G_n^{p,q}(1,x) = [n-1]_{p,q}\sum_{k=1}^{\infty} s_{n,k}^{p,q}(x)p^{(n-1)^2+k}q^{k(k-1)}\int_0^{\infty/A} s_{n,k-1}^{p,q}(t)d_{p,q}t + s_{n,0}^{p,q}(x)$$

$$= [n-1]_{p,q}\sum_{k=1}^{\infty} s_{n,k}^{p,q}(x)p^{(n-1)^2+k}q^{k(k-1)}\begin{bmatrix} n+k-2 \\ k-1 \end{bmatrix}_{p,q}\int_0^{\infty/A} \frac{t^{k-1}}{(1\oplus x)_{p,q}^{n+k-1}}d_{p,q}t + s_{n,0}^{p,q}(x)$$

$$= [n-1]_{p,q}\sum_{k=1}^{\infty} s_{n,k}^{p,q}(x)p^{(n-1)^2+k}q^{k(k-1)}\begin{bmatrix} n+k-2 \\ k-1 \end{bmatrix}_{p,q}\frac{B_{p,q}(k,n-1)}{p^{(n-1)/2}q^{k/2}} + s_{n,0}^{p,q}(x)$$

$$= \sum_{k=0}^{\infty} p^{\frac{n}{2}}q^{\frac{k}{2}}s_{n,k}^{p,q}(px) = 1.$$

(ii) For $f(t) = t$, we have

$$G_n^{p,q}(t,x) = [n-1]_{p,q}\sum_{k=1}^{\infty} s_{n,k}^{p,q}(x)p^{(n-1)^2+k}q^{k(k-1)}\int_0^{\infty/A} s_{n,k-1}^{p,q}(t)t\,d_{p,q}t$$

$$= [n-1]_{p,q}\sum_{k=1}^{\infty} s_{n,k}^{p,q}(x)p^{(n-1)^2+k}q^{k(k-1)}\begin{bmatrix} n+k-2 \\ k-1 \end{bmatrix}_{p,q}\int_0^{\infty/A} \frac{t^{k}}{(1\oplus t)_{p,q}^{n+k-1}}d_{p,q}t$$

$$= [n-1]_{p,q}\sum_{k=1}^{\infty} s_{n,k}^{p,q}(x)p^{(n-1)^2+k}q^{k(k-1)}\begin{bmatrix} n+k-2 \\ k-1 \end{bmatrix}_{p,q}\frac{B_{p,q}(k+1,n-2)}{p^{(n-2)/2}q^{(k+1)/2}}$$

$$= \frac{1}{[n-2]_{p,q}}\sum_{k=1}^{\infty} s_{n,k}^{p,q}(x)\frac{p^{(n-1)^2+k}q^{k(k-1)}}{p^{\frac{n-2}{2}}q^{\frac{k+1}{2}}}[k]_{p,q}$$

$$= \frac{[n]_{p,q}x}{pq[n-2]_{p,q}}\sum_{k=0}^{\infty} p^{(n+1)/2}q^{k/2}s_{n+1,k}^{p,q}(px) = \frac{[n]_{p,q}x}{pq[n-2]_{p,q}}.$$

(iii) When $f(t) = t^2$, we get

$$G_n^{p,q}(t^2,x) = [n-1]_{p,q}\sum_{k=1}^{\infty} s_{n,k}^{p,q}(x)p^{(n-1)^2+k}q^{k(k-1)}\int_0^{\infty/A} s_{n,k-1}^{p,q}(t)t^2d_{p,q}t$$

$$= [n-1]_{p,q}\sum_{k=1}^{\infty} s_{n,k}^{p,q}(x)p^{(n-1)^2+k}q^{k(k-1)}\begin{bmatrix} n+k-2 \\ k-1 \end{bmatrix}_{p,q}\int_0^{\infty/A} \frac{t^{k+1}}{(1\oplus t)_{p,q}^{n+k-1}}d_{p,q}t$$

$$= [n-1]_{p,q}\sum_{k=1}^{\infty} s_{n,k}^{p,q}(x)p^{(n-1)^2+k}q^{k(k-1)}\begin{bmatrix} n+k-2 \\ k-1 \end{bmatrix}_{p,q}\frac{B_{p,q}(k+2,n-2)}{p^{(n-3)/2}q^{(k+2)/2}}$$

$$= \frac{1}{[n-2]_{p,q}[n-3]_{p,q}}\sum_{k=1}^{\infty} s_{n,k}^{p,q}(x)\frac{p^{(n-1)^2+k}q^{k(k-1)}}{p^{\frac{n-3}{2}}q^{\frac{k+2}{2}}}[k]_{p,q}[k+1]_{p,q}$$

$$= \frac{[n]_{p,q}x^2}{p^2q^4[n-2]_{p,q}[n-3]_{p,q}}\sum_{k=0}^{\infty} p^{(n+2)/2}q^{k/2}s_{n+2,k}^{p,q}(px)$$

$$= \frac{[2]_{p,q}[n]_{p,q}x}{p^{-n+4}q^3[n-2]_{p,q}[n-3]_{p,q}}\sum_{k=0}^{\infty} p^{(n+1)/2}q^{k/2}s_{n+1,k}^{p,q}(px)$$

$$= \frac{[n]_{p,q}x^2}{p^2q^4[n-2]_{p,q}[n-3]_{p,q}} + \frac{[n]_{p,q}[2]_{p,q}x}{p^{-n+4}q^3[n-2]_{p,q}[n-3]_{p,q}}.$$

Lemma 2. For $0 < q < p \leq 1$, we have the following explicit formulae for the central moments

(i) $G_n^{p,q}((t-x),x) = \frac{(1-pq)[n]_{p,q}x+[2]_{p,q}xpq}{pq[n-2]_{p,q}}$

(ii) $G_n^{p,q}((t-x)^2,x) = \left(\frac{[n]_{p,q}[n+1]_{p,q}}{p^2q^4[n-2]_{p,q}[n-3]_{p,q}} - \frac{[n]_{p,q}[2]_{p,q}}{pq[n-2]_{p,q}} + 1\right)x^2 + \frac{[n]_{p,q}[2]_{p,q}x}{p^{-n+4}q^3[n-2]_{p,q}[n-3]_{p,q}}$.

Remark 1. For $0 < q < 1$and $q < p \leq 1$, we may have that $\lim_{n\to\infty}[n]_{p,q} = \frac{1}{(q-p)}$.
To find the convergence of (p,q)-Baskakov-Durrmeyer operators, we consider $p = p_n$ and$q = q_n$are such that $0 < q_n < p_n \leq 1$ andfor sufficiently large n, $p_n \to 1$, $q_n \to 1$, $p_n^n \to a$, $q_n^n \to b$and $[n]_{p_n,q_n} \to \infty$.

3. Main Results

Definition 1: Let $C_{x^2}[0,\infty)$, be the class of all function *f*, which are defined on the positive real axis and satisfy$|f(x)| \leq C(1+x^2)$, where C is a positive constant depending on *f*. By $C_{x^2}[0,\infty)$, we mean, the subspaceof all functions $f \in C_{x^2}[0,\infty)$, for which $\lim_{|x|\to\infty}\frac{f(x)}{1+x^2}$ is finite. The class $C_{x^2}^*[0,\infty)$is endowed with the norm

$$\|f\|_{x^2} = \sup_{x\in[0,\infty)} \frac{|f(x)|}{1+x^2}.$$

Theorem 1. Let $p = p_n$ and $q = q_n$ satisfy $0 < q_n < p_n \leq 1$ and for sufficiently large n, $p_n \to 1$, $q_n \to 1$, then for each $f \in C_{x^2}^*[0,\infty)$, we have

$$\lim_{n\to\infty}\left\|G_n^{p_n,q_n}(f) - f\right\|_{x^2} = 0.$$

Proof. By ([7], Theorem 4.1.4), it is sufficient to show that

$$\lim_{n\to\infty}\left\|G_n^{p_n,q_n}(t^k,x) - x^k\right\|_{x^2} = 0,\ k = 0,1,2. \quad \textbf{(3.1)}$$

Since $G_n^{p_n,q_n}(1,x) = 1$, the first condition of (3.1) is satisfied for *k = 0*.
In view of Lemma 1, we have

$$\left\|G_n^{p_n,q_n}(t,x) - x\right\|_{x^2} = \sup_{x\in[0,\infty)} \frac{\left|G_n^{p_n,q_n}(t,x) - x\right)|}{1+x^2}$$
$$\leq \left(\frac{[n]_{p_n,q_n}}{p_nq_n[n-2]_{p_n,q_n}} - 1\right) \sup_{x\in[0,\infty)} \frac{x}{1+x^2} \leq \left(\frac{[n]_{p_n,q_n}}{p_nq_n[n-2]_{p_n,q_n}} - 1\right).$$

Taking limit on both the sides as $n \to \infty$,

$$\lim_{n\to\infty}\left\|G_n^{p_n,q_n}(t,x) - x\right\|_{x^2} = 0.$$

The condition of (3.1) is satisfied for *k*=1.
Again using Lemma 1, we obtain

$$\left\|G_n^{p_n,q_n}(t^2,x) - x^2\right\|_{x^2} = \sup_{x\in[0,\infty)} \frac{\left|G_n^{p_n,q_n}(t^2,x) - x^2\right)|}{1+x^2}$$
$$\leq \left(\frac{[n]_{p_n,q_n}[n+1]_{p_n,q_n}}{{p_n}^2{q_n}^4[n-2]_{p_n,q_n}[n-3]_{p_n,q_n}} - 1\right) \sup_{x\in[0,\infty)} \frac{x^2}{1+x^2}$$
$$+ \left(\frac{[n]_{p_n,q_n}[2]_{p_n,q_n}}{{p_n}^{-n+4}{q_n}^3[n-2]_{p_n,q_n}[n-3]_{p_n,q_n}}\right) \sup_{x\in[0,\infty)} \frac{x}{1+x^2}$$
$$\leq \left(\frac{[n]_{p_n,q_n}[n+1]_{p_n,q_n}}{{p_n}^2{q_n}^4[n-2]_{p_n,q_n}[n-3]_{p_n,q_n}} - 1\right) + \left(\frac{[n]_{p_n,q_n}[2]_{p_n,q_n}}{{p_n}^{-n+4}{q_n}^3[n-2]_{p_n,q_n}[n-3]_{p_n,q_n}}\right),$$

which implies that

$$\lim_{n\to\infty}\left\|G_n^{p_n,q_n}(t^2,x) - x^2\right\|_{x^2} = 0.$$

Thus the proof is completed.

Definition 2: Let $C_B[0,\ \infty)$ be the space of all real valued uniformly continuous and bounded function *f* on the interval$[0,\ \infty)$. For $f \in C_B[0,\ \infty)$ the Peetre's K-functional is defined as

$K_2(f,\delta) = inf\{\|f-g\| + \delta\|g''\|; g \in C_B^2[0,\infty)\}$.

where $\delta > 0$ and $C_B^2[0,\infty) = \{g \in C_B[0,\ \infty); g', g'' \in C_B[0,\ \infty)\}$. By Devore and Lorentz [6], there exists anabsolute constant *P*> 0 such that

$$K_2(f,\delta) \le P\omega_2\left(f,\sqrt{\delta}\right),$$

where the second order modulus of continuity is defined as

$$\omega_2\left(f,\sqrt{\delta}\right) = \sup_{0<|h|\le\sqrt{\delta}} \sup_{0\le x\le\infty} |f(x+2h) - 2f(x+h) + f(x)|,$$

and the usual modulus of continuity is given by

$$\omega(f,\delta) = \sup_{0<|h|\le\sqrt{\delta}} \sup_{0\le x\le\infty} |f(x+h) - f(x)|.$$

Theorem 2. Let $f \in C_B[0,\ \infty)$ and $x \ge 0$, then there exists a constant $P > 0$, such that

$$\left\|G_n^{p_n,q_n}(f,x) - f(x)\right\| \le P\omega_2\left(f,\sqrt{\delta_n^{p_n,q_n}(x)}\right),$$

where

$$\delta_n^{p_n,q_n}(x) = \left(\frac{[n]_{p_n,q_n}[n+1]_{p_n,q_n}}{{p_n}^2{q_n}^4[n-2]_{p_n,q_n}[n-3]_{p_n,q_n}} - \frac{[n]_{p_n,q_n}[2]_{p_n,q_n}}{p_nq_n[n-2]_{p_n,q_n}} + 1\right)x^2 + \frac{[n]_{p_n,q_n}[2]_{p_n,q_n}x}{{p_n}^{-n+4}{q_n}^3[n-2]_{p_n,q_n}[n-3]_{p_n,q_n}}.$$

Proof. Let $g \in C_B^2[0,\infty)$, then by Taylor's expansion

$$g(t) = g(x) + g'(x)(t-x) + \int_x^t (t-k)g''(k)dk,\quad t \in [0,\infty),$$

which implies that

$$G_n^{p_n,q_n}(g,x) - g(x) = G_n^{p_n,q_n}\left(\int_x^t (t-k)g''(k)dk; x\right).$$

Hence

$$\left|G_n^{p_n,q_n}(g,x) - g(x)\right| \le G_n^{p_n,q_n}(t-x)^2\|g''\|$$
$$= \left[\left(\frac{[n]_{p_n,q_n}[n+1]_{p_n,q_n}}{{p_n}^2{q_n}^4[n-2]_{p_n,q_n}[n-3]_{p_n,q_n}} - \frac{[n]_{p_n,q_n}[2]_{p_n,q_n}}{p_nq_n[n-2]_{p_n,q_n}} + 1\right)x^2 + \frac{[n]_{p_n,q_n}[2]_{p_n,q_n}x}{{p_n}^{-n+4}{q_n}^3[n-2]_{p_n,q_n}[n-3]_{p_n,q_n}}\right]\|g''\|.$$

Also by Lemma 1, we have

$$\left|G_n^{p_n,q_n}(f,x)\right| \le \|f\|.$$

Therefore, we have

$$\left|G_n^{p_n,q_n}(f,x) - f(x)\right| \le \left|G_n^{p_n,q_n}(f-g,x) - (f-g)(x) + \left|G_n^{p_n,q_n}(g,x) - g(x)\right|\right|$$
$$\le 2\|f-g\| + \left[\left(\frac{[n]_{p_n,q_n}[n+1]_{p_n,q_n}}{{p_n}^2{q_n}^4[n-2]_{p_n,q_n}[n-3]_{p_n,q_n}} - \frac{[n]_{p_n,q_n}[2]_{p_n,q_n}}{p_nq_n[n-2]_{p_n,q_n}} + 1\right)x^2 + \frac{[n]_{p_n,q_n}[2]_{p_n,q_n}x}{{p_n}^{-n+4}{q_n}^3[n-2]_{p_n,q_n}[n-3]_{p_n,q_n}}\right]\|g''\|.$$

Taking the infimum on the right hand side over all $g \in C_B^2[0,\infty)$and applying the Peetre's *K*-functional, we get the required result.

$$\left\|G_n^{p_n,q_n}(f,x) - f(x)\right\| \le P\omega_2\left(f,\sqrt{\delta_n^{p_n,q_n}(x)}\right).$$

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